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Trying to Make a Lever Work at Ages 1 to 4: The Development of "Functions" (Logico-Mathematical Thinking)

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To find out if children could make *functions* before age 4, 73 children aged 1 to 4 were encouraged to imitate the use of a lever to make a beanbag fly up. Functions are mental relationships that preoperational children can make between 2 things at a time in a unidirectional way (Piaget, Grize, Szeminska, & Bang, 1968/1977). The child's construction of the following 3 functions was hypothesized and confirmed: (a) As a function of being pushed down, the up end of the board (the lever) goes down; (b) as a function of this descent, the down end of the board goes up; and (c) as a function of this ascent of the board, the beanbag flies up. Three developmental levels were found, and educational implications are discussed.

Until 1968, Piaget wrote about preoperational children's logic mostly in negative terms as a time when children cannot yet conserve, cannot yet class include, cannot yet seriate, and so on. In *Epistemology and Psychology of Functions* (Piaget, Grize, Szeminska, & Bang, 1968/1977), however, he evaluated one type of

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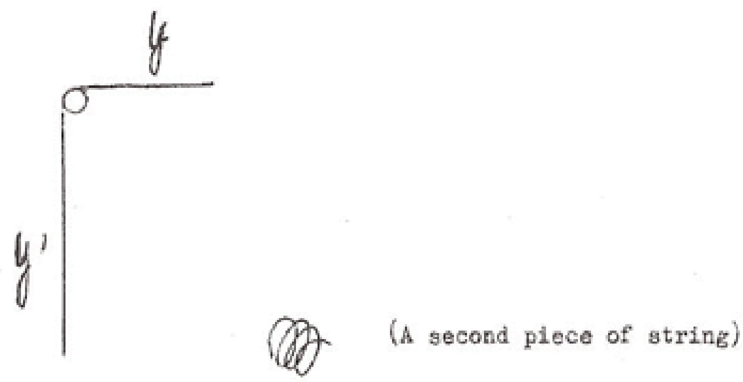

FIGURE 1 The material used to find out about the presence of (a) functions and (b) conservation.

preoperational logic positively and called it *functions*. Functions are mental relationships that preoperational children create between two things at a time in a unidirectional way.

For example, in individual interviews (Piaget, 1977), preoperational children were presented with a string draped across a nail as shown in Figure 1. Another piece of string was then cut to have the same length as the first one, and the child was asked to ascertain that the two pieces of string had the same length. The second piece of string was then rolled up and set aside, and the child was asked what would happen if the upper end of the string (y) was pulled to the right. The preoperational child correctly predicted that y would become longer and that y' would become shorter. The string was then pulled to the right, and the child was asked if the string $y + y'$ was just as long as the second piece of string that had been rolled up. The child's answer was no. This is an example of a function, a relationship preoperational children can make between two things (y and y') at a time in a unidirectional way, before becoming able to conserve length. These children do not think about the possibility of pulling y' back to its original position.

The experiment described in the preceding paragraph was included in another study that was published in *Epistemology and Psychology of Functions* (Piaget et al., 1968/1977). A simplified version of the apparatus used in the later study is presented in Figure 2. In this apparatus, the string ($y + y'$) was attached to a spring, x, which was attached to a wall, and the child could pull the string down at z. When the children were asked what would happen if z was pulled down, the first relationship the Level I children made at ages 4 through 6 was between the action of pulling z and the spring stretching. The Level I children said, in other words, that the spring, x, would become longer as a function of the string being pulled down at z. Most of the Level I children thus overlooked the intermediary variables between the action of pulling the string at z and its effect on the spring. At Level II, between 7 and 10 years of age, however, the children coordinated the intermediary pairs and

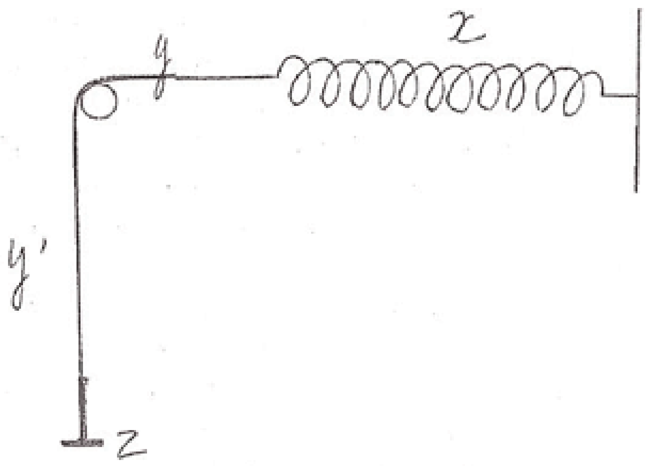

FIGURE 2 The apparatus used to find out about children's coordination of functions.

said that pulling z would make y' longer (a function between z and y'), that y' would in turn pull y and make it shorter (a function between y' and y), and that y would then pull x and make it longer (a function between y and x). As they coordinated these intermediary pairs with increasing precision and reasoned that the increase in y' is compensated by a decrease in y and that a decrease in y is compensated by an increase in x, the children later became able to conserve the length of $y + y'$ and of $y + x$.

In *Epistemology and Psychology of Functions* (Piaget et al., 1968/1977), Piaget gave many examples of functions in the logico-mathematical realm but presented only the aforementioned experiment that involved physical knowledge. To understand the difference between the logico-mathematical and physical realms, it is necessary to review the fundamental distinction Piaget (1967/1971a) made between physical and logico-mathematical knowledge according to their ultimate sources. *Physical knowledge* is knowledge of objects in the external world. Knowing that a spring will stretch when it is pulled is an example of physical knowledge. The ultimate source of physical knowledge is objects in the external world, but the source of *logico-mathematical knowledge* is in each individual's head. When a person is presented with a red block and a blue one, for example, and says that the two blocks are different, "different" is an example of a mental relationship that person has made in his or her head. The red block and the blue one are observable (physical knowledge), but the difference between them is not (logico-mathematical knowledge). The difference is made by each person who thinks about the two blocks as being different. If the same person decides to ignore color, the same blocks can become "similar" or "the same." If, on the other hand, the person decides to think about the blocks numerically, the two blocks can become "two." Logico-mathematical knowledge consists of the mental relationships that each individual makes, and its ultimate source is in each individual's mind.

In the preceding discussion about physical and logico-mathematical knowledge, we spoke as if the two kinds of knowledge exist separately in the human mind. In reality, however, according to Piaget, the two kinds of knowledge exist together, inseparably, until the age of 5 or 6. At this age, logico-mathematical knowledge begins to be differentiated from physical knowledge (as can be seen in the conservation of number). However, physical knowledge always remains dependent on logico-mathematical knowledge. For example, to find out that pulling a spring makes it longer, one has to be able to make the logico-mathematical relationship of "longer." To figure out that pulling the string with twice as much force makes the spring stretch 1.3 times as much, for example, one needs more elaborate logico-mathematical knowledge. Figuring out the numerical relationship between two variables belongs to physics—the logico-mathematization of physical knowledge. The function between y and x (y becomes shorter as a function of x becoming longer) is also an example of the logico-mathematization of physical knowledge.

In the experiment involving a spring in *Epistemology and Psychology of Functions* (Piaget et al., 1968/1977), the youngest children were 4 years old. We wanted to know if functions could be experimentally demonstrated at a younger age in an activity involving physical knowledge. Therefore, we decided to ask 1- to 4-year-olds to imitate the use of a lever. In this task, the interviewer put a beanbag on the down end of a lever (see Figure 3a) and quickly pushed the up end down to make the beanbag fly up. We chose this task partly because we knew that it appealed to 4- and 5-year-olds (Kamii & DeVries, 1978/1993) and partly because we saw the following three functions in it:

1. As a function of being pushed down, the up end of the board goes down.
2. As a function of this descent, the down end of the board goes up.
3. As a function of this ascent of the board, the beanbag flies up.

We chose this task also because we had found in a previous study (Miyakawa, Kamii, & Nagahiro, 2005) that a good way to get 1- to 3-year-olds to comply with a request is to encourage them to imitate an intriguing action. We also knew from previous research (Kamii & DeVries, 1978/1993) that these kinds of physical-knowledge activities attract babies and young children. Physical-knowledge activities are those in which children act on objects, mentally and physically, to produce a desired effect.

In sum, the purpose of the present study was to find out if 1- to 4-year-olds make and coordinate functions (logico-mathematical knowledge) in a physical knowledge activity. Aside from the study reported by Piaget et al. (1968/1977), the only mention of functions in a physical-knowledge activity found in the literature is a brief reference to it by DeVries, Zan, Hildebrandt, Edmiaston, and Sales (2002). Although the development of logic in infancy has been studied by re-

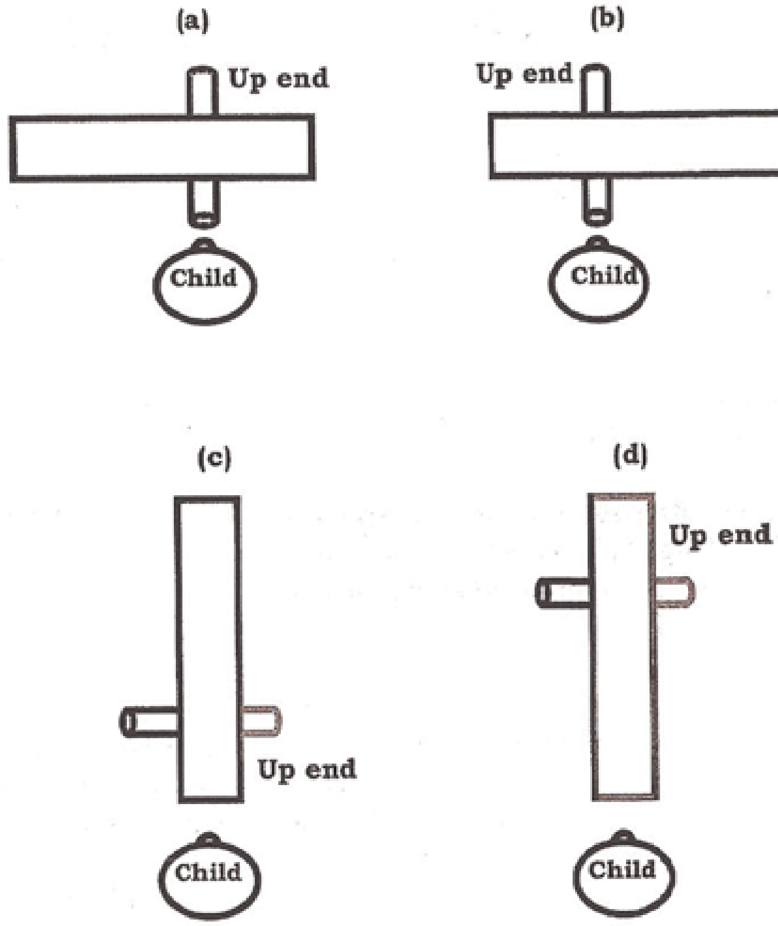

FIGURE 3 The arrangements made in Procedures 4a, 4b, 4c, and 4d.

searchers such as Langer (1980) and Sinclair, Stambak, Lezine, Rayna, and Verba (1982/1989), no work can be found about functions in the first 2 years of life. We wanted to know when functions could be observed before age 4.

METHOD

The participants were 73 children attending one of three private child care centers in Okayama City and Fukuyama City, Japan. Because both parents had to be employed for a child to qualify for day care, the sample can be characterized as middle class with both parents working. These sites were selected because their teachers worked together to develop a constructivist curriculum. See Table 1 for age and gender distributions; note that in the table, 1:0 means 1 year and 0 months.

TABLE 1: Age and Gender Distributions of the 73 Children

Age	*Total*	*Boys*	*Girls*	*Average Age*
1;0 to 1;5	10	5	5	1 year, 3 months
1;6 to 1;11	12	7	5	1 year, 8 months
2;0 to 2;5	10	3	7	2 years, 3 months
2;6 to 2;11	11	6	5	2 years, 8 months
3;0 to 3;5	10	4	6	3 years, 3 months
3;6 to 3;11	10	4	6	3 years, 8 months
4;0 to 4;5	10	5	5	4 years, 2 months
	73	34	39	

Note. 1;0 = 1 year, 0 months.

The materials used were a wooden board (12 × 60 cm, 0.5 cm thick), a paper tube (30 cm long, diameter of 5 cm), and a beanbag (made of cloth, 6 × 6 × 6 cm, weighing 30 g). All of the interviews were videotaped.

Procedure

1. The interviewer brought the child to the game room (or sometimes stayed in the child's classroom when he or she was reluctant to go) and sat on the floor with the child. (A game room in a Japanese child care center is like a small gym with a stage at one end.)

2. The interviewer put the board on the tube to make a lever and said to the child, "Watch what I'm going to do, because I'm going to make this beanbag fly up in the air." The interviewer then placed the beanbag on the down end of the board, making sure the child was watching, and quickly pushed the up end down to make the beanbag fly up.

3. If the child seemed interested in a suggestion to imitate this action, the interviewer proceeded to Procedure 4a. If not, she repeated the demonstration three to seven times (depending on the child's degree of indifference), trying to coax him or her. If the child showed interest, the interviewer proceeded to Procedure 4a. If not, the session was terminated.

4a. Saying, "Would you like to try it?," the interviewer handed the beanbag to the child, keeping the lever in the same position as before (Figure 3a). If the child succeeded in making the beanbag fly up three or more times in succession, the interviewer proceeded to Procedure 4b. If not, the demonstration was repeated as many times as necessary, and the interview was terminated when the child's actions could be characterized.

4b. Saying, "Let's change the board like this," the interviewer changed the lever to the position in Figure 3b, making sure the child was watching. With the up end of the board now on the opposite side, the interviewer handed the beanbag to the child. If the child succeeded in making the beanbag fly up three or more times in succession, the interviewer went on to Procedure 4c. If not, the interview was ended when the child's actions could be characterized.

4c. Making sure that the child was watching, the interviewer changed the position of the board as shown in Figure 3c so that the up end would be directly in front of the child and the down end would be away from him or her. "I wonder if you can make the beanbag fly up when I change the board like this," the interviewer said. If the child was successful three or more times in succession, the interviewer went on to Procedure 4d. Otherwise, the session was ended when the child's actions could be characterized.

4d. Making sure that the child was watching, the interviewer changed the lever to the position in Figure 3d. Moving the up end of the board away from the child, she asked, "I wonder if you can make the beanbag fly up when I change the board like this." The interview was terminated when the child's actions could be characterized.

The preceding steps describe only the outline of the procedure. In reality, a clinical interview requires additional interventions depending on each child's responses. For example, if the child did not apply sufficient force in pushing the board down, the interviewer sometimes pushed the board down with the child. Remarks such as "I wonder what's a good place to put it [the beanbag]" were also made to focus the child's attention, when appropriate.

RESULTS

The three of us viewed the videotapes together several times to see if developmental levels could be conceptualized. Generally, we first agreed on a specific criterion (or criteria) and then counted its frequency. For example, after agreeing to define Level 0 as one where the child showed no interest in the activity, we counted the number of times each child picked up the beanbag and put it on the board. If the frequency was zero, the child was categorized at Level 0. If it was more than zero, the child was categorized at Level I or higher.

The Developmental Levels Found

The four levels agreed on in the end are summarized here and described later in more detail.

Level 0. Showing no interest in the activity.
Level I. Acting only on the beanbag (no function made).
 Type IA. Picking up the beanbag and putting it on the board.
 Type IB. Putting the beanbag on the board and hitting it directly.

Level II. Acting on the board but being unable to make the beanbag fly up (only one function made).

Level III. Acting on the board and making the beanbag fly up (all three functions made).

Our criteria for categorizing a child at Level I were that the child (a) put the beanbag on the board and (b) never hit the board. Our criteria for categorizing a child at Level II were that the child (a) put the beanbag on the board and (b) hit the board. The Level II criteria did not specify where on the board the beanbag was to be placed or which part of the board the child was to hit. For Level III, our criteria were that the child (a) always put the beanbag on the down end of the board in all four of the positions illustrated in Figures 3a, 3b, 3c, and 3d; and (b) always hit the up end of the board. We were able to make judgments of never and always probably because of the presence or absence of functions. When a child had not made the first function (as a function of being pushed down, the up end of the board goes down), it did not occur to a Level I child to hit the board. Once a child made the first function at Level II, however, it did not occur to him or her to act on anything except the board. Likewise, when a child had made all three functions, it became obvious to him or her that the place to put the beanbag was on the down end of the board and that the place to hit was its up end.

Level 0: Showing no interest in the activity. The children categorized at Level 0 were the 4 youngest children, who were 1;0, 1;0, 1;0, and 1;3.

Example 1. Riku (1;0)

The interviewer repeatedly made the beanbag fly up while asking Riku to watch her, but Riku did not even look at the beanbag. The expression on his face remained unchanged, and the only time he showed any interest in the beanbag was when it landed on the floor near him and when the interviewer offered it to him. He then picked it up, lifted it, and dropped it or threw it.

Level I: Acting only on the beanbag (no function made). Type IA babies (Example 2) merely put the beanbag on the board and showed no reaction when coaxed repeatedly to make it fly up. By contrast, Type IB babies (Example 3) put the beanbag on the board and hit it directly. Type IB children used the relationship Type IA children made and acted directly on the beanbag rather than indirectly through an action on the board. Types IA and IB were considered types rather than sublevels because they overlapped in age. Type IA was observed from 1;3 to 2;0, and Type IB was found from 1;4 to 1;8.

Example 2. Type IA, Yamato (1;5)

Yamato showed interest by smiling as soon as the beanbag flew up and pointing to it when it landed on the floor. After seeing a demonstration sev eral times, he even began to anticipate the beanbag's flight by looking at the down end of the board as soon as the interviewer touched the up end. However, whenever the beanbag was handed to him with encouragement to make it fly up, he placed it on the board and showed no intention of doing anything else.

Example 3. Type IB, Tomoki (1;8)

Tomoki was like Yamato (Example 2), but he hit the beanbag after placing it on the board. In spite of many repeated remarks and demonstrations, like "I wonder why it doesn't fly up. Let me show you again," Tomoki put the beanbag randomly at various places on the board and always hit it directly.

Because the Level I children did not act on the board, they cannot be said to have made even the first function we hypothesized (as a function of being pushed down, the up end of the board goes down). By contrast, at Level II children began to hit or push the board down, thereby manifesting the belief that they could make the beanbag fly up by acting on the board.

Level II: Acting on the board but being unable to make the beanbag fly up (only one function made). The progress at Level II was that the child now always tried to make the beanbag fly up by acting on it indirectly (by acting on the board). But if he or she succeeded in making the beanbag fly up, this success was the result of random trial and error. Level II extended over the widest age range, from 1;9 to 3;9, and had the greatest number of children. Within this level, it was impossible to identify sublevels because only 2 children clearly succeeded in Procedure 4a but not in Procedure 4b, and only 2 clearly succeeded in Procedure 4b but not in Procedure 4c.

Within Level II, however, one kind of progress was clearly found: Children quickly figured out that the end of the board to push down was the up end. This relationship between the action on the board and its up end corresponded to the first function we hypothesized—as a function of being pushed down, the up end of the board goes down. As can be seen in Table 2, this first function appeared for the first time at the age of 1;10. Of the 29 children at Level II, 24 pushed the end of the board that was up at least 75% of the time. (To categorize every child's placement of the beanbag and every action on the board, we made the three categories of (a) the side of the board that is up, (b) the side that is down, and (c) the middle [defined as within 10 cm of the line along which the board was resting on the tube]. After tallying every child's every action together, we calculated percentages and made Table 2.)

At Level II, it was easy for children to decide which end of the board to push down but very difficult to figure out where to put the beanbag. Of the Level II children, 21% (6 out of 29, as can be seen in Table 3) put the beanbag in the middle where the board was resting on the tube at least 75% of the time. Because the

TABLE 2
Level II Children Who Pushed the Up End of the Board Down at Least 75% of the Time

Age	*Pushing the Up End of the Board at Least 75% of the Time*
1;9	
1;10	x
1;10	x
2;0	
2;1	
2;1	x
2;2	x
2;2	x
2;4	x
2;4	x
2;4	x
2;5	x
2;6	
2;6	x
2;7	x
2;7	x
2;7	x
2;7	x
2;7	x
2;8	x
2;11	x
2;11	x
3;0	x
3;4	x
3;4	
3;6	x
3;7	x
3;9	x
3;9	x

beanbag placed there sometimes jumped a little if the child pushed the board forcefully and quickly enough, it was often impossible for him or her to see that this was an undesirable place. Probably the most accurate way to describe what Level II children did with the beanbag is to say that they placed it randomly at various places on the board. Some of them sometimes put the beanbag on the up end of the board and pushed the same end down.

Within Level II, significant signs of progress observed among some older 3-year-olds were hesitation, self-correction, and immobility with both hands up while trying to decide which end of the board to push down. For example, Miyu (3;9) put the beanbag on the up end of the board, got ready to push the same end

TABLE 3
Level II Children Who Put the Beanbag Over the Tube at Least 75% of the Time

Age	*Putting the Beanbag Over the Tube at Least 75% of the Time*
1;9	
1;10	x
1;10	
2;0	x
2;1	x
2;1	
2;2	
2;2	
2;4	
2;4	x
2;4	
2;5	
2;6	
2;6	
2;7	
2;7	
2;7	x
2;7	
2;7	
2;8	
2;11	
2;11	
3;0	
3;4	
3;4	x
3;6	
3;7	
3;9	
3;9	

down, but interrupted herself to move the beanbag to the down end. Conflict was also evident when a child put one hand out to hit the board, then the other hand, and kept both hands up while trying to decide which end of the board to act on. These behaviors were manifestations of emerging functions. Miyu interrupted herself probably because the second and third functions were emerging. She got ready to push the up end of the board down, but it then occurred to her that the beanbag could not fly up if it was on the same end of the board that she was about to push down. When she thus coordinated all three functions, she "saw" that as a function of the up end of the board going down, the down end would go up and make the beanbag go up. When she thus coordinated the three functions, she felt the need to move the beanbag to the down end of the board.

Level III: Acting on the board and making the beanbag fly up (all three functions made). Level III was easy to identify. Once children figured out why they should push the up end of the board down and why the beanbag should be placed on the opposite end, they had no difficulty making the beanbag fly up in any of the four situations (Figures 3a, 3b, 3c, and 3d). When the children solidly coordinated the three functions we hypothesized, all four of the positions became easy to deal with. In other words, Level II children sometimes achieved success by trial and error, but they could not reproduce their success because they did not understand why they were successful. Level III children were consistently successful because they knew the reason for their success.

In sum, the first function was easy even for 2-year-olds to make, but the second and third functions took a long time to emerge. The reason is probably that the second and third functions required the making of relationships that went counter to the first function. It was easy even for 2-year-olds to "see" that they should push the up end of the board but impossible for them to think about what would happen to the down end. As can be seen in Table 4, only 65% of the 3-year-olds made the second and third functions. However, 100% of the 4-year-olds made all three functions, and making a lever work became easy for all of them.

The Relation Between the Developmental Levels and Children's Ages

As can be seen in Table 4, there was a clear relationship between children's ages and the levels found. Levels 0 and I were found mostly among the 1-year-olds; Level II, among the 2-year-olds; and Level III, among the 4-year-olds. The relationship between these levels and the three functions we hypothesized is discussed in the next section.

TABLE 4
Relation Between Developmental Levels and Ages

	Level				
Age	*0*	*I*	*II*	*III*	*Total*
1;0–1;11	4 (18%)	15 (68%)	3 (14%)		22
2;0–2;11		1 (5%)	19 (90%)	1 (5%)	21
3;0–3;11			7 (35%)	13 (65%)	20
4;0–4;5				10 (100%)	10
Total	4	16	29	24	73

DISCUSSION

The purpose of this study was to find out whether children younger than age 4 make and coordinate the following three functions:

1. As a function of being pushed down, the up end of the board goes down.
2. As a function of this descent, the down end of the board goes up.
3. As a function of this ascent of the board, the beanbag flies up.

We found that 76% of the 2-year-olds made the first function (16 2-year-olds at Level II, according to Table 2, divided by 21), and 65% of the 3-year-olds made all three of these functions (see Table 4). All of the 4-year-olds made all three of these functions.

The first function was easy for Level II children to make, but the second and third functions took a long time to emerge. The second function was especially hard to make, probably because it went in a direction opposite than the first. When children concentrated on pushing the board down to make it go down, thinking about the opposite end going up was probably impossible. When older Level II children began to show hesitation, indecision, and self-corrections, this indicated that the children were beginning to make new relationships in their minds. Until this time, the Level II children had been busy putting the beanbag anywhere on the board by trial and error, but they now began to feel a contradiction between putting the beanbag on the up end of the board and pushing the same end of the board down. When this happened, the children began to move the beanbag to the down end of the board.

At Level III, the children made and coordinated all three of the functions, and how to make the beanbag fly up became obvious. When this happened, the activity became boring to the children. All three of the functions we hypothesized can thus be said to have emerged toward the end of Level II.

It was gratifying that the data confirmed the three functions we had hypothesized. However, by rereading the chapter about the spring and the string mentioned at the beginning of our article (Piaget et al., 1968/1977, and Figure 2), we became aware that we had overlooked an initial, global function that later differentiated into the three functions we had hypothesized. The first function reported by Piaget et al. was between the children's action of pulling the string at z and the end result of the spring stretching (x). The three intermediary functions—(z, y'), (y', y), and *(y, x)—emerged later.*

We returned to the data to see if we might find a similar first function and indeed found one at Level II. At Level I, the children in our study acted directly on the beanbag, but at Level II they started to act on the board to make the beanbag fly up. The Level II children acted on the board because they expected this action to result in the beanbag's ascent. This relationship the Level II children made between their action and the end result was the first undifferentiated function the children made, and the three functions we hypothesized were the intermediary functions they made later. The 5 Level II children without an x in Table 2 (i.e., those who did not push the up end of the board at least 75% of the time) were the ones who made this global function but not the first function we had hypothesized.

This study illustrates an important point Piaget made: Human beings see things not only with their eyes but also with their minds (Bringuier, 1977/1980). When they could not "see" that she was acting on the board, the Level I children imitated the interviewer incorrectly by acting directly on the beanbag. Level I children's logico-mathematical relationships (functions) were not developed sufficiently to "see" that the interviewer was acting indirectly on the beanbag by acting on the board. Likewise, Level II children could not "see" for a long time that the interviewer was putting the beanbag on the down end of the board. No matter how many times we demonstrated how to make the lever work, most Level II children could not "see" that the interviewer was always placing the beanbag on the down end of the board. These examples illustrate human beings' dependence on logico-mathematical knowledge to acquire physical knowledge.

Piaget (1965/1971b) described the relationship between human beings' acquisition of physical and logico-mathematical knowledge in the following way:

> Starting [in infancy] from a state of centration on a self uncognizant of itself and in which the subjective and objective are inextricably intermingled, the progressive decentration of the subject leads to a twofold movement, of externalization, tending to physical objectivity, and internalization tending to logico-mathematical coherence. But physical knowledge remains impossible without the logico-mathematical framework and it is impossible to construct the latter without its being applicable to "any" object whatever. (p. 115)

The present study demonstrated how a child becomes able to "see" the interviewer's action on the up end of the board when he or she makes the first function (logico-mathematical knowledge). This study also showed that a child becomes able to "see" the interviewer's placement of the beanbag on the down end of the board when he or she makes the second and third functions. Physical knowledge is indeed impossible to construct without a logico-mathematical framework.

Piaget (1937/1954, p. 355) also described the interrelated development of physical and logico-mathematical knowledge with the two circles shown in Figure 4. In this figure, A is the point where the child meets the most superficial part of the environment and all analogous points. Y is the pole of externalization tending toward the acquisition of physical knowledge, and X is the pole of internalization tending toward the elaboration of logico-mathematical knowledge. The more elaborate the child's logico-mathematical knowledge becomes, the more he or she can "see" in the world of objects, and vice versa. For example, when the Level II child con-

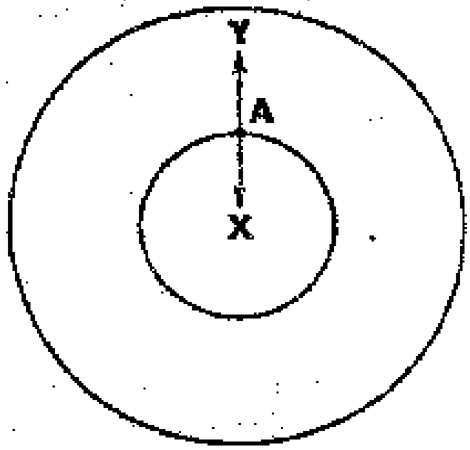

FIGURE 4 The child's development of logico-mathematical knowledge (X) and physical knowledge (Y). A is the point where the child meets the most superficial part of the environment and all analogous points.

structed the first function we had hypothesized (logico-mathematical knowledge), he or she became able to "see" that the interviewer was pushing the up end of the board. By imitating this action, the child acquired a considerable amount of physical knowledge. Out of the physical knowledge acquired by pushing the up end of the board, it became possible for the child to construct the second and third functions (logico-mathematical knowledge). Logico-mathematical knowledge indeed makes the acquisition of physical knowledge possible, but the new physical knowledge thus gained makes possible the further elaboration of logico-mathematical knowledge.

Educational Implications

The educational implication of this study is that physical-knowledge activities like the one involving a lever are good for young children's development of logico-mathematical knowledge. Physical-knowledge activities are especially good for young children's intellectual development, because they motivate children to try to produce a desired effect, and children can tell immediately whether they were successful. The immediate feedback from objects motivates children to think hard to figure out how to modify their actions. As Piaget said, "It is when events or phenomena must be explained and goals attained through an organization of causes that operations [logico-mathematical relationships] will be used [and developed] most" (Piaget, 1971/1974, p. 17).

The development of logico-mathematical knowledge in physical-knowledge activities has been documented in Kamii, Miyakawa, and Kato (2004); Miyakawa et al. (2005); and Kamii, Rummelsburg, and Kari (2005). The third study is especially significant because it showed that physical-knowledge activities can encourage low-performing, low-socioeconomic status first graders to think hard, thereby building the cognitive foundation necessary for arithmetic. The present study further attests to the thinking children do, especially toward the end of Level II. It was pointed out that they stop their actions when they make new mental relationships, hesitate, and think about what to do differently to produce the desired effect. Logico-mathematical thinking develops when children think, and the present study revealed specific mental relationships that preoperational children can make and coordinate.

When the activity used in the present study becomes too easy, the teacher can introduce a longer board and ask children if they can jump on it to make a big beanbag fly up to the ceiling (see Kamii & DeVries, 1978/1993, chapter 5). Children will then begin to vary the distance between the beanbag and the fulcrum, and between the fulcrum and the upper end of the board, trying to see if these factors make a difference to the height of the beanbag's flight.

Two of the most important principles of teaching when encouraging children to think are (a) to refrain from suggesting what to do and from reinforcing "correct" behaviors, and (b) to let children decide how long they want to play. Traditional teaching has taught teachers to reinforce correct responses, but in a physical-knowledge activity children can tell whether or not they were successful. A teacher's reinforcement is not only superfluous but also likely to interfere with the child's thinking. Likewise, any suggestion from an adult about how to be successful deprives children of an opportunity to do their own thinking. The objective in a physical-knowledge activity is not that children learn how to make a lever. The objective is for children to have opportunities to make mental relationships.

If children want to continue playing with a lever, this is proof that they are intrinsically motivated to think. If they are motivated, they will want to struggle for a long time. If they are not, their time is better spent playing with something else. Many other examples of physical-knowledge activities can be found in *Physical Knowledge in Preschool Education* (Kamii & DeVries, 1978/1993). The reader is encouraged to use and modify these kinds of activities to encourage children to make and coordinate functions and to develop their logico-mathematical knowledge.

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